Correlation of High-Pressure Arc Heater Results

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A method is introduced to correlate data from high-pressure arc heaters. It is suited for those heaters which have losses dominated by radiation. The correlation is valid for both laminar and turbulent flows. Turbulence is postulated to effect only the thermal conduction, and not electrical conduction or radiation. The introduction of azimuthal velocity (i.e., swirl) to the flow can stabilize the arc at the center. An axial decay of swirl will result in loss of stability and secondary flows. The scaling parameters have also been applied to vortex stabilized arcs. A method for scaling heaters is introduced, using the developed scaling parameters.

Nomenclature

B_z = axial magnetic field
D = diameter
E = electric field
F_1 = inlet length parameter
F_2 = inlet voltage parameter
F_3 = inlet power loss parameter
f = parametric variable used in inlet theory
H = channel width
h_{av} = average enthalpy in the heater
I = arc current
j = current density
L = heater length
$\dot{m}$ = mass flow rate in the constrictor
Nu = Nusselt number
P = arc power = IV
P_{cons} = power loss to walls
(p/p_a) = pressure in atmospheres
$(q_{av})_R$ = average wall heat flux due to radiation = $(P_{cons})_R/\pi DL$
q_W = design value for wall heat flux
R = wall radius
Re = Reynolds number
r = radial coordinate
r_1 = radius of arc
u = radial velocity; power radiated per unit volume
V = arc voltage
v = azimuthal velocity
w = axial velocity
w' = fluctuating component of axial velocity
y = distance from wall to channel
z = axial coordinate
η = efficiency
κ = thermal conductivity
μ^* = viscosity at reference temperature
ρ = density
σ = electrical conductivity
ϕ = thermal conduction potential = dT
ψ = sonic flow parameter

$\langle d\sigma/d\phi\rangle$, $\langle du/d\phi\rangle_a$, $\langle dh/d\phi\rangle$ etc., are plasma property approximations. For example, the specific radiated power is given by $u = \langle du/d\phi\rangle_a(p/p_a)\phi$.

Introduction

HIGH-power plasma arc heaters have been extensively studied for use in test facilities for re-entry simulation. Two types will be considered in this paper. The first is a wall constricted type using a segmented constrictor with axial flow. The second is the Lindè type, which has a solid conducting wall in which the arc is vortex stabilized. These are shown schematically in Fig. 1. During the past ten to fifteen years, scientifically advanced technology has significantly improved constricted arc heaters.[1–5] Empirical procedures have led to a Linde heater design which has operated at over 200 atmospheres pressure, has rugged electrodes that have operated at over 5 ka of current, and has supported over 15 kv across one insulator.[6–9] There is a great deal of sense in attempting to develop the science of this type of device, to understand the performance limits, and eventually optimize the design.

A great deal of the theory developed for constricted arcs is applicable to the Linde heater. However, several important new phenomena need to be introduced into the analysis before it can be adequately applied. The first of these is the effects of the high Reynolds number of the flow. The flow will almost certainly be turbulent. The effect that this will have on the thermal conduction processes needs to be carefully investigated. More important, the strong vortex plays a dominant role in the Linde arc heater. The vortex can have many effects on the velocity and pressure distribution throughout the heater, all of which need at least a preliminary investigation. A few of the most important effects could be: 1) A vortex has been shown to stabilize an arc. As the vortex decays, it is possible that the arc may become unstable at some axial position downstream from the region of gas injection. This onset of instability may determine the arc length in the heater. 2) As the vortex strength is increased, the axial velocity along the centerline has been shown to decrease.[10,11] At some point, the axial velocity reverses and secondary flows set in. These effects have strong influences on the convective gas heating rate and radial heat conduction, that need to be carefully analyzed.

In this paper, some data for the two types of heaters will be compared in terms of inlet parameters. This will lead to a design procedure for scaling vortex stabilized, radiation dominated arc heaters.

1. Laminar Inlet Theory

The heating of a gas by an arc in a coaxial configuration has been studied extensively in the laminar regime.[12–16] Quite good agreement between theory and experiment has been obtained. The theory incorporates an energy balance, continuity and some phenomenological relationships (Ohm's Law, heat conduction and heat radiation). This theory will form a starting point, to be expanded by adding turbulence.

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A. Equations

The dissipated power heats the gas and is conducted to the wall or is radiated,

$$jE = \rho w \partial h/\partial z - (1/r)\partial(r\partial\phi/\partial r)/\partial r + u \tag{1}$$

In Eq. (1) there are many assumptions including no radial flow, potential depends upon z only, laminar heat conduction, no mass addition, optically thin radiation, etc. The Ohm's Law $j = \sigma E$ can be integrated to give

$$I = E \int_0^{r_1} \sigma 2\pi r dr \tag{2}$$

In Eq. (2) there is no contribution for $r > r_1$, since σ is effectively zero. The energy Eq. (1) can be integrated over the cross section to give

$$IE = \dot{m}(dh_{\text{av}}/dz) + dP_{\text{cons}}/dz \tag{3}$$

where (neglecting net axial radiation)

$$dP_{\text{cons}}/dz = 2\pi R(-\partial\phi/\partial r)_{r=R} + \int_0^{r_1} u 2\pi r dr \tag{4}$$

Finally, the integral over the length

$$IV = \dot{m}\Delta h_{\text{av}} + P_{\text{cons}} \tag{5}$$

B. Plasma Transport Properties

In order to integrate Eq. (1) to solve for the radial and axial dependence of the enthalpy, it is necessary to first know the relationships between σ, h, ϕ, and u. Two methods commonly used are to choose the best known functional relationships and use numerical integration, or to choose approximate linear relationships and analytically integrate. Figure 2 shows the important electronic properties plotted as u vs σ.

C. Choice of Parameters

The purpose of the analysis here is to find some appropriate combination of the variables for correlating data from existing heaters. The method to choose the variables is to first solve the linearized equations, and then keep the same dimensional groups as appear in the theoretical solutions. Hence, we will use the linearizing technique. Let $\sigma = \langle d\sigma/d\phi\rangle$, $h = \langle dh/d\phi\rangle \phi$, etc. The inlet theory has been integrated using the Bessel model for the radial distributions, and can be summarized in terms of the inlet parameter f.

$$L = z = \frac{\dot{m}(dh/d\phi)}{2\pi R^2[(2.4/R)^2 + (du/d\phi)]} \ln\frac{1}{1-f^2} \tag{6}$$

$$h_{\text{av}} = \frac{I\langle dh/d\phi\rangle}{\pi R^2(d\sigma/d\phi)^{1/2}[(2.4/T)^2 + (du/d\phi)]^{1/2}} f \tag{7}$$

$$E = \frac{[(2.4/R)^2 + (du/d\phi)]^{1/2}}{(d\sigma/d\phi)^{1/2}} \cdot \frac{1}{f} \tag{8}$$

$$dP_{\text{cons}}/dz = \frac{I[(2.4/R)^2 + (du/d\phi)]^{1/2}}{(d\sigma/d\phi)^{1/2}} \cdot f \tag{9}$$

$$V = \frac{\dot{m}\langle dh/d\phi\rangle}{2\pi R^2(d\sigma/d\phi)^{1/2}[(2.4/R)^2 + (du/d\phi)]^{1/2}} \ln\frac{1+f}{1-f} \tag{10}$$

$$P_{\text{cons}} = \frac{\dot{m}I\langle dh/d\phi\rangle}{2\pi R^2(d\sigma/d\phi)^{1/2}[(2.4/R)^2 + (du/d\phi)]^{1/2}} \left(\ln\frac{1+f}{1-f} - 2f\right) \tag{11}$$

where $f = 0$ at inlet and $f \to 1$ as $z \to \infty$.

In an arc with radiation dominated losses, $(du/d\phi) \gg (2.4/R)^2$. Since $(du/d\phi)$ is approximately proportional to the pressure in the enthalpy range of interest, we write $(du/d\phi) = (du/d\phi)_a(p/p_a)$. From Eq. (6), we see that

$$F_1 = R^2(p/p_a)L/\dot{m} \tag{12}$$

is a measure of the inlet length where the numerical factors and transport properties have been deleted.

From Eqs. (10) and (6), it follows that for small z, V varies as $z^{1/2}$. Divide Eq. (10) by the square root of Eq. (6) to get a quantity independent of z for $z \to 0$. Taking out the numerical constants and plasma properties, we are left with

$$F_2 = RV/(\dot{m}L)^{1/2} \tag{13}$$

F_2 is an inlet voltage parameter. From Eqs. (11) and (6), it can be seen that for small z, P_{cons} varies as $z^{3/2}$. Divide Eq. (11) by the three half power of Eq. (6) to get a quantity independent of z for small z. Assume that radiation dominates conduction, and we get a heat conduction parameter

$$F_3 = \dot{m}^{1/2}(P_{\text{cons}})_R/RL^{3/2}I(p/p_a) \tag{14}$$

It has been found convenient to choose another form which is F_2 times F_3, or

$$F_2F_3 = (P_{\text{cons}})_R V/IL^2(p/p_a) \tag{15}$$

From Eqs. (6, 10, and 11), it can be seen that F_2F_3 varies only 33% as the length goes from zero to infinity.

An enthalpy parameter, which comes from Eqs. (6) and (7) by combining to find a form independent of L as $L \to 0$, and then dropping the plasma term is

$$\frac{\dot{m}^{1/2}Rh_{\text{av}}}{L^{1/2}I} = \frac{RV}{\dot{m}^{1/2}L^{1/2}} - \frac{R^2L(p/p_a)}{\dot{m}} \frac{\dot{m}^{1/2}(P_{\text{cons}})_R}{L^{3/2}RI(p/p_a)} \tag{16}$$

Equation (16), which relates four correlation parameters is the exact energy equation. Since the enthalpy parameter can be expressed in terms of the other three, no further examination is needed. [The variables F_1, F_2, and F_3 will be used for correlating experimental data. If the plasma properties were linear, then the exact relationships could be found from Eqs. (6, 10, and 11). Here we will find the relationships from experimental results.]

Another method of finding parameters for the correlation of data is through the process of dimensional analysis. Such an approach is outlined by Yas'ko.[17] Reference values of enthalpy, thermal conductivity, radiation per unit volume, and electrical conductivities are introduced into the list of variables. This resulting set of variables can be rearranged to give three groups which contain the dimensional variables of

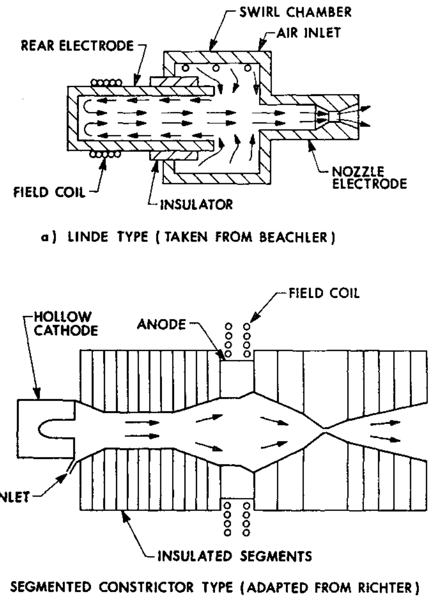

Fig. 1 Schematic of heater types.

F_1, F_2, and F_3, plus some other groups. Yas'ko arrives at groups by strictly nondimensionalizing the equations, while the present method uses a crude linearized solution of the equations, first to then define the groups in terms of the approximate solutions. This results in a smaller set of parameters. Thus, what we have done is to try to eliminate such variables as L/R, and Mach number by clever choice of the variables that are kept. What is gained is a smaller set of variables. What is lost is a universally applicable set of correlations. By omitting the viscous terms in the momentum equation, the Reynolds number was lost, which then lost the parameter needed to correlate the effects of turbulence. Yas'ko does not introduce a power loss parameter, such as a Stanton or Nusselt number which is needed for complete comparisons, and heater performance. F_3 is the heat transfer parameter of this paper. Another thing done here is to approximate that the only place that pressure explicitly appears in the transport properties is in the power radiated. While this is not precisely true, it is approximations of this type which allow for simplified correlations. Thus, the present method is much more powerful than a general approach of nondimensionalizing equations.

D. Optimum Temperature Profiles

The plasma in the arc should be kept as cool as possible, consistent with the desired gas enthalpy. Because of the steep dependence of radiation upon temperature, a very hot thin arc will radiate more than a fat cooler arc with the same average enthalpy in the heater. This flat profile also reduces the electrical conductivity, thus maintaining high electric field. The enthalpy profile may be controlled by changing turbulent intensity. This might be accomplished by swirl.

2. Turbulence

Turbulence is produced in arc heaters run at high Reynolds numbers. An alternate method to induce turbulence seems to be by producing an instability associated with an applied axial magnetic field. The turbulence will affect the energy transfer processes in the arc. Here the mechanisms of turbulence will be analyzed, so they can be added to the above theory.

A. Reynolds Number

In pipe flow, it is customary to use the Reynolds number based upon diameter, $\rho w(2R)/\mu$. This can alternately be expressed in terms of the total mass flow as $2\dot{m}/\pi R\mu$. The trouble comes in evaluating the viscosity in a stream which has a large variation of properties. A viscosity μ^* will be used in this paper, which corresponds to 6000°K, which is between the wall and peak temperatures, and is hoped to be representative of the boundary layer

$$Re = 2\dot{m}/\pi R\mu^* \tag{17}$$

Arc heaters operating at high pressures and high power will usually operate at high Reynolds number, and tend to be turbulent. This can be seen by using a sonic exit condition to determine the radius at the sonic throat.

$$\dot{m} = \psi_h \pi R_t{}^2 p/h_{\text{av}}{}^{1/2} \tag{18}$$

and an efficiency to determine the mass flow

$$\dot{m} = \eta P/h_{\text{av}} \tag{19}$$

Combining

$$Re = (4\psi_h \eta p P/\pi)^{1/2}(\mu^*)^{-1}h_{\text{av}}{}^{-3/4}(R_t/R) \tag{20}$$

For similar geometries and enthalpies, the Reynolds number is proportional to the square root of the product of pressure and power.

B. Transport Properties

It is expected that thermal conduction and momentum transfer will be directly affected by turbulence, while electrical conduction and the volume power radiated will only be indirectly affected. The usual method to account for turbulence is to introduce an eddy thermal conductivity or a mixing length. The turbulent heat flux is given by

$$q_{\text{turb}} = -C_p \rho l^2 |\partial w/\partial r| \partial T/\partial r \tag{21}$$

Approximate $\partial w/\partial r = 2w/R$, and using continuity to get $w = \dot{m}/\rho\pi R^2$

$$q_{\text{turb}} = -(l/R)^2(2\dot{m}/\pi R)\partial h/\partial r. \tag{22}$$

The mixture length, l, is usually of order of $\frac{1}{10}$[18] of the radius R. Combining with the laminar term

$$q = -[1 + (l/R)^2 RePr]\partial\phi/\partial r \tag{23}$$

Equation (23) should be valid for low degree of ionization where most of the energy is carried by heavy particles. The only effect of dissociation upon turbulent transfer is through the effect upon enthalpy.

Viscosity is unimportant since most pressure drop is due to heating, not friction. The electrical conductivity (σ) and the specific power radiated u are primarily electronic phenomena. They can be expressed in terms of the temperature and pressure for the case of equilibrium dissociation and ionization. The "turbs," or small zones moving about rapidly in the turbulent flow, are so large that they must be electrically neutral, (i.e., their typical dimension is much larger than a Debye length). Thus, turbulence does not affect transport electrical charge. The presence of turbulence in an arc can, of course, modify the temperature, and thus effect electrical conductivity indirectly. Radiation is due to acceleration of charges. Since the accelerations of the "turbs" are small, there will be no additional radiation due directly to turbulence. Thus, Fig. 2 is valid for both laminar and turbulent cases, and the only change needed to account for turbulence is the modifications of the heat-transfer term.

C. Wall Heat Flux in the Turbulent Case

For turbulent flows, the wall heat transfer due to radiation and to turbulent conduction will have a similar dependence upon pressure. The radiant heat flux is given by

$$q_{\text{rad}} = \frac{1}{2\pi R}\int_0^{r_1} u2\pi r dr \tag{24}$$

Since u is approximately proportional to pressure, let $u = u_a(p/p_a)$

$$q_{\text{rad}} = R(p/p_a)\int_0^{r_1} u_a(r/R)(dr/R) \tag{25}$$

Thus, for similar arcs

$$q_{\text{rad}} \sim R(p/p_a)$$

The turbulent heat conduction can be found from Eq. (21) which, for similar arcs $q_{\text{turb}} \sim \dot{m}/R^2$. Due to the sonic exit condition, the mass flow will increase with the product $\dot{m} \sim R^2(p/p_a)$, hence both turbulent heat conduction and wall heat flux due to radiation tend to increase linearly with pressure for similar heaters, and will be hard to distinguish from variations in heat transfer with mass flow.

3. Swirl

Azimuthal velocity is often used in the art of arc heater design. The gas enters through inlets with an angle to the axial direction.

A. Arc Stabilization

This swirl stabilizes the arc by centrifuging the less dense hot plasma to the center of the heater. The only stable configuration is with the hot, and hence, electrically conducting, gas along the axis. This discourages arc attachment to the wall for the axial locations where there is swirl. As the swirl decays, the stability is lost, and the arc goes to the wall. There is evidence that in the Linde heater, the arc length without swirl is zero. The arc length may be the same as swirl decay length.

B. Secondary Flows

Secondary flows are introduced by the decay of azimuthal velocity between axial locations along the heater. A radial pressure drop is introduced by the swirl

$$\partial p/\partial r = \rho v^2/r \tag{26}$$

If v decays with z, then

$$\partial(\partial p/\partial z)/\partial r = \partial(\partial p/\partial r)/\partial z < 0 \tag{27}$$

Thus the axial pressure drop ($\partial p/\partial z$ drives the axial flow) is more negative at larger radius. In the usual case of secondary flow, the axial velocity is decreased along the centerline, as in Fig. 3. With enough decay, this can even cause the axial flow to reverse.

This secondary flow would be undesirable for arc heaters since there is a tendency for the hottest gas to remain in the heater, and the cooler layers to exit. Fortunately, in most heaters this does not occur since swirl induced pressure drop (and, hence, also its decay) is greatly reduced by the heating. The heating reduces ρ in Eq. (26). The value of v is ordinarily limited by a sonic flow condition for the inlet of cold gas.

C. Heat Transfer

Swirl may have a direct effect upon turbulent heat transfer. Two possible mechanisms are: 1) The centrifugal buoyancy of the lighter "turbs" will make them move toward the center, thus decreasing the outward flow of heat. This is analogous to the heating of a liquid from above which is a stabilizing influence, and 2) The swirl itself may increase the intensity of the turbulence near the center of the constrictor, as in Fig. 2.

4. Correlation of Experimental Data

The parameters introduced above will be evaluated for results known for two types of arc heaters. When correcting for

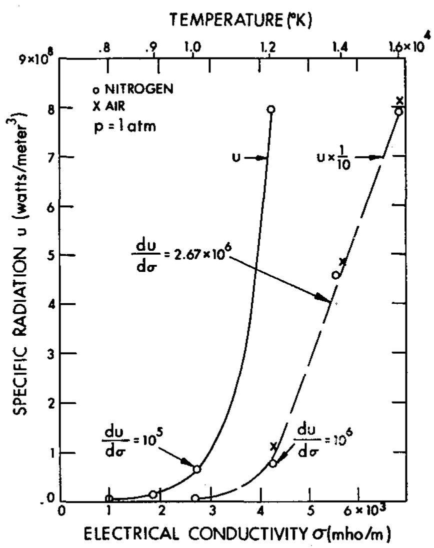

Fig. 2 Specific radiation vs electrical conductivity.

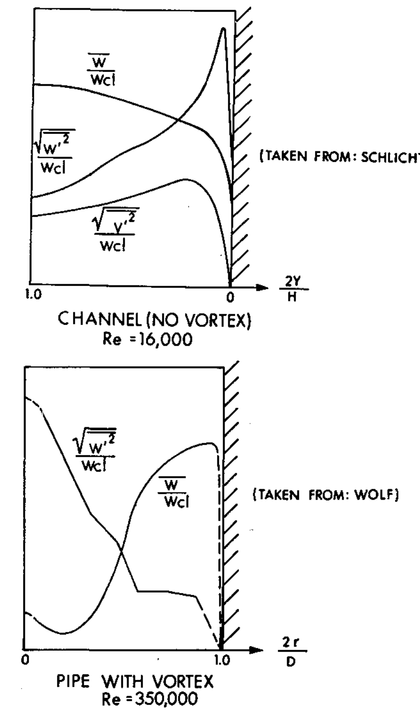

Fig. 3 Effect of swirl upon turbulence and secondary flow.

turbulence, conduction has the same dependence as radiation, hence no change is needed in F_1 to F_3. Constricted heater data is unpublished, but the heater is of the high-pressure type.[19] For the Linde type heater, only the data for which the arc length is believed known were chosen. The data is selected from Smith.[9] These cover a wide range of mass currents and pressures.

A. Radiation-Conductivity Correlation

The produce F_2F_3 was chosen so that it should depend only upon electronic plasma properties, $F_2F_3 \approx \langle du/d\sigma \rangle_a$, and should thus be independent of Reynolds number (i.e., turbulence). Figure 4 shows that the value is reasonably constant for each type of heater. The lower value for the high Reynolds number points could be explained by one of two mechanisms. First, there could be a different temperature, and hence $\langle du/d\sigma \rangle_a$. This would occur (see Fig. 2) if the Linde heater had a lower temperature. Second, it may be that the swirl of the Linde heater reduced the turbulent heat conduction as suggested in Section 3 C.

B. Heat-Transfer Correlation

The heat-transfer data, which is a result of radiation and turbulent conduction, will be compared with the values expected if there is only turbulent heat transfer. This is done

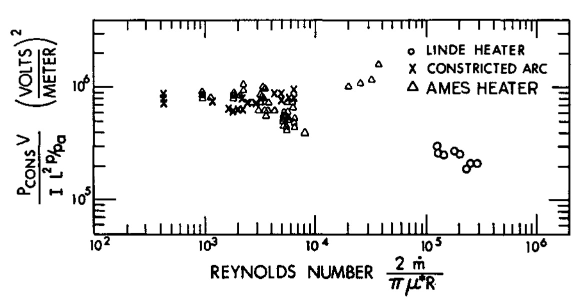

Fig. 4 Radiation-electrical conductivity correlation vs Reynolds number.

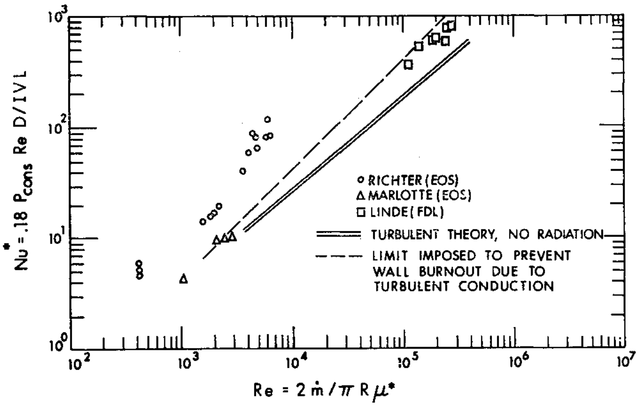

Fig. 5 Heat-transfer correlations.

on a plot of a conduction parameter vs Reynolds number in Fig. 5.

$$Nu^* = (2R)q_{\mathrm{wall}}/\kappa(IV/\dot{m}C_p) \tag{28}$$

Thus, the quantity plotted is a ratio of actual average wall heat flux to that which a laminar flow would have if the temperature were $IV/\dot{m}C_p$. Expressing mass flow in terms of Reynolds number

$$Nu^* = (\mu^* C_p/4\kappa)P_{\mathrm{cons}}\ \mathrm{Re}(2R)/(IV)L \tag{29}$$

It appears that in the data, there is more radiation than turbulent conduction.

The effects of turbulence upon the radiation from the arcs has not been measured at high pressure. A low-pressure study using argon in a constricted arc yields some significant results which are of interest.[4] The results are significant since both cases have small arcs in the center of a constrictor. The radiation results are shown in Fig. 6. At low Reynolds number the radiation was the same at both locations. At higher Reynolds number there was a marked decrease at the downstream location. The difference is likely due to the fact that the argon was introduced with swirl which stabilized the arc near the inlet. The reduction was due to gas dynamic turbulence which mixes the arc with the gas, producing a broader-cooler region with less total radiation.

Experimental data from a constricted heater is correlated for heat transfer in terms of $F_3/2\pi$ vs F_1 in Fig. 7. There is considerable scatter for small F_1; however, a definite trend is noted. The linear theory would have predicted F_3 is constant for small F_1 and F_3 is inversely proportional to the square root of F_1 for large F_1. For the Linde type heater, using the heat transfer to the front electrode, the data for $0.5 < F_1 < .15$ shows F_3 is approximately a constant equal to 1200, in MKSA units.

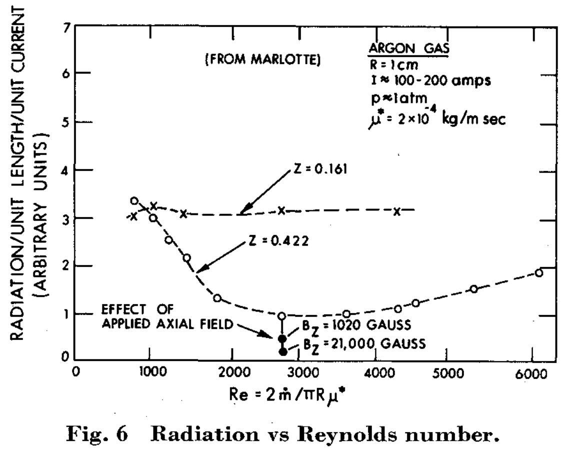

Fig. 6 Radiation vs Reynolds number.

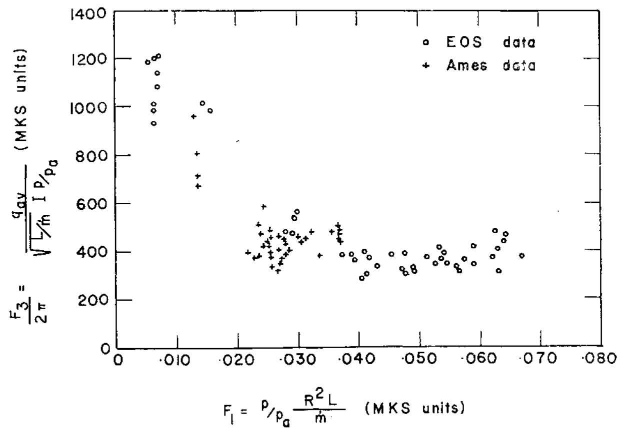

Fig. 7 Correlation of wall heat flux data.

C. Voltage Correlations

Experimental data from the constricted heater for the voltage parameter is correlated in Fig. 8. Since $F_2 \cdot F_3$ is nearly constant, this figure appears like the reciprocal of Fig. 7. The Linde heater data is almost constant with F_2 equal to 220.

5. Scaling Law Summary

The variables which have been introduced and correlated can be used as the basis of a heater design. The analysis of an arc heater usually starts with radius, length, current, mass flow and pressure and solves for wall heat flux, voltage, enthalpy, efficiency, etc. For design, we wish to specify enthalpy, pressure, power, wall heat flux, and then solve for radius, length, current, voltage, efficiency. The scatter in the correlations may be due to experimental scatter or inadequacy of the theoretically chosen parameters to collapse the data. The data should be adequate for design estimates.

The method is as follows: 1) choose P, h_{avg}, p/p_a and a value of F_1, 2) evaluate F_2 from Fig. 8, 3) evaluate F_3 from Fig. 7,

$$4)\quad \eta = 1 - F_1F_3/F_2,\ \text{and} \tag{30}$$

5) determine the value of wall heat flux that must be designed for

$$(q_{\mathrm{av}})_R \leq (F.S.)\left(\frac{h_{\mathrm{av}}p/p_aF_3}{6\pi\eta F_2}\right)^{2/3}\left(\frac{P}{40\pi}\right)^{1/3} \tag{31}$$

when $(F.S.)$ is a factor of safety. This equation is derived on the basis that the wall heat flux is twice that due to radiation,

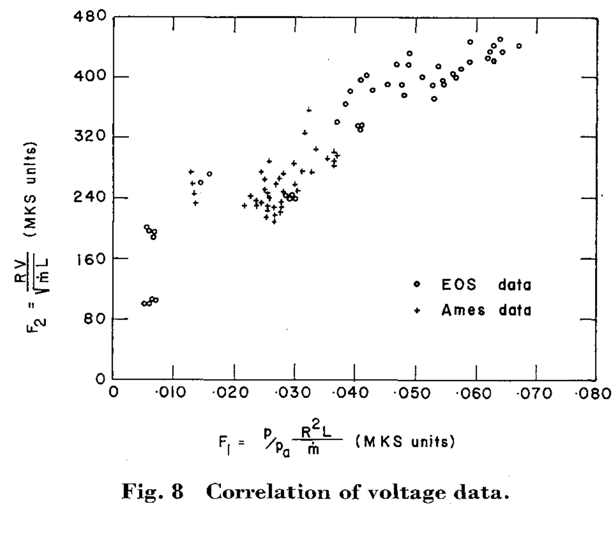

Fig. 8 Correlation of voltage data.

hence

$$(q_W) \geq 2(q_{av})_R$$

$$6) \quad \dot{m} = \eta P/h_{av} \tag{32}$$

$$7) \quad L = (1 - \eta)^2 P(p/p_a)h_{av}/(2\pi)^2\eta(q_{av}{}^2)_R F_1 \tag{33}$$

$$8) \quad R = 2\pi\eta(q_{av})_R F_1/(1 - \eta)(p/p_a)h_{av} \tag{34}$$

$$9) \quad V = (1 - \eta)^2 P h_{av}[F_2(p/p_a)/F_1]^{3/2}/(2\pi)^2 \times \eta(q_{av}{}^2)_R(F_2)^{1/2} \tag{35}$$

$$10) \quad I = P/V \tag{36}$$

Steps 1–10 represent a closed form algorithm to get a design.

6. Conclusions and Recommendations

1) A data correlation parameter $VP_{\text{cons}}/IL^2(p/p_a)$ has been introduced which should be valid for radiation dominated arcs. It has been found to be constant over a wide range of operation.

2) Decay of swirl can cause undesirable secondary flows. Addition of mass along the constrictor, with the purpose of maintaining (or increasing) swirl can enhance the flow along the centerline of the hottest gas.

3) Swirl may have a direct effect to reduce outward turbulent heat transfer in an arc heater by centrifugally confining hot gas. This should be further investigated.

4) The length of the arc is effected by the decay of swirl.

5) A heater design method has been introduced in terms of correlating parameters F_1 to F_3.

Appendix I: Distributed Mass Injection

The designers of some segmented arc heaters have found it desirable to inject the gas more or less uniformly over some "inlet" length, L, of the arc heater. This has been done, for instance, by Charles Shepard at NASA-Ames. When this injection technique is employed, some correction to the scaling parameters must be made. An approximate method of doing this is to perturb the solution for injection at one end. When this is done, the scaling parameters are modified as follows:

$$F_1 = F_1$$

$$F_2 = [RV/(\dot{m}L)^{1/2}]1/[1 + \tfrac{1}{2}(L_1/L)^{1/2}]$$

$$F_3 = (\dot{m}^{1/2}P_{\text{cons}}/RL^{3/2}Ip/p_a)1/[1 + \tfrac{1}{2}(L_1/L)^{3/2}]$$

These expressions indicate that for a heater of fixed length and radius operated at the same mass flow rate, current, and pressure, then if the mass is injected uniformly along the length rather than all at one end, the voltage would be reduced by a factor of 2 and the power absorbed by the wall would be increased by a factor of $\frac{3}{2}$.

Data obtained by Charles Shepard in the Ames constricted heater has been kindly supplied by Dr. Velvin Watson. In correlating this data, the correction factors for distributed injection shown above have been used with a value of $L_1 = L/2$.

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